...IONAL HURRICANE RESEARCH PROJECT

REPORT NO. 25

Distribution of Surface Friction
in Hurricanes

U. S. DEPARTMENT OF COMMERCE
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by

Lester F. Hubert

U. S. Weather Bureau, Washington, D. C.

NATIONAL HURRICANE RESEARCH PROJECT REPORTS

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DISTRIBUTION OF SURFACE FRICTION IN HURRICANES

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ABSTRACT

Terms of the horizontal equations of motion are evaluated from surface analysis of three hurricane and three non-hurricane cases in order to investigate the components of friction (tangential to and normal to the anemometer wind direction) in the boundary layer. A method of moisture flux analysis is developed to study the variation of height of the inflow layer which in turn yields a clue concerning the variation of stress profile in time and space.

While the magnitude of friction increases toward the hurricane center it is clear that it is not a simple function of wind speed or surface roughness. It is also shown that, (1) the component of friction normal to the surface wind direction is almost always significant, (2) accelerations are quite small and are not correlated with the downstream pressure force, (3) accelerations and angle of cross-isobar flow are correlated, and (4) total friction is not correlated with angle of cross-isobar flow.

It is suggested that a compensating mechanism characterizes the boundary layer which changes the angle of cross-isobar flow (thus changing the downstream pressure force) to compensate a tendency toward acceleration brought about by changes of momentum flux from upper layers as well as changes of surface stress. This compensation in turn provides a variable component of friction normal to the surface wind which is almost always significant.

1. INTRODUCTION

The loss of momentum of the atmosphere through the mechanism of stress at the earth's surface is the most important sink of kinetic energy in hurricanes and for that reason investigation of the energy budget will be restricted until quantitative data are obtained either by direct measurement or are deduced from other parameters.

Investigation both in wind tunnels and on the meteorological scale has shown that surface stress (τ_o) is given by an equation of the form,

$$\tau_o = \rho \; c_d^2 \; V^2 \tag{1}$$

where V is the total wind speed, ρ density of the air, and c_d a drag coefficient essentially constant for a given surface roughness (sometimes a coefficient of friction $C = c_d^2$ is used, e. g., in [3]). Thus while it is true that stress is a function of surface parameters, the effect of stress on the boundary layer of air is not so determined. In fact the drag on the air is largely unknown because the acceleration is proportional to the shear of stress (mostly with altitude) instead of to the stress itself, as is shown by the following equations of horizontal motion:

$$\frac{dV}{dt} = -\frac{1}{\rho} \frac{\partial p}{\partial s} + \frac{1}{\rho} \frac{\partial \tau_t}{\partial z} \tag{2}$$

$$\frac{V^2}{R} = -fV - \frac{1}{\rho} \frac{\partial p}{\partial n} + \frac{1}{\rho} \frac{\partial \tau_n}{\partial z} \tag{3}$$

where the s-axis is oriented downstream, the n-axis normal to the left of the s-axis, and τ_t, τ_n components of the stress along those axes.[1]

Obtaining values for surface stress requires vertical integration of (2) and (3) by means of methods used by Charnock et al. [1] or by Palmén and Riehl [5], and it will be shown later that this exceeds the capability of the data used. The aim here however is to study the effect of stress on the air, not stress itself. The boundary layer is decelerated by a continual vertical transport of momentum which is proportional to the derivative of stress, therefore the non-integrated forms of equations (2) and (3) are used to determine to what extent the behavior of the fluid in two dimensions implies the characteristics of the pertinent forces in the third dimension, insofar as their effect on momentum transport is concerned.

The primary purpose then, is to study the distribution of the term $\frac{1}{\rho} \frac{\partial \tau}{\partial z}$ at anemometer level to determine (a) if there is a systematic variation of this frictional effect with distance from the center, and (b) whether or not this effect can be associated with some other, more easily measured, parameters of surface circulation.

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[1]Four other components of the stress vector which depend upon horizontal shear have been omitted here because it is safe to assume that these terms are one or two orders of magnitude smaller than the terms included. This may not be valid at the wall of the hurricane eye, but that region is not considered here.

It is clear that analysis of a single horizontal plane can lead to conclusions concerning a three-dimensional layer that are at best, only inferential. In order to get some further insight into the nature of the vertical variation of the boundary layer, a method is developed to estimate the depth of the inflow layer. Even partial results are of value in this area where so little is known because they point up problems that require more investigation and warn us from using invalid models in other research concerning the boundary layer.

2. FRICTION TERM ANALYSIS

Surface data for Hazel 1954, Connie 1955, Diane 1955, and three non-hurricane cases, March 25, 1947, March 27, 1950, and March 11, 1952, were analyzed to permit evaluation of each of the terms in equations (2) and (3) with the exception of the last terms on the right[2] which were then obtained by summing all of the measured quantities (table 1).

Figure 1. - Track of three hurricanes and area of analysis.

To reduce the number of variables that could influence the vertical eddy terms, all analysis was restricted to the same geographical area (the portion of our southeastern coast shown in figure 1) and the situations were selected so that the range of wind speeds was about the same for all cases. With moderately high winds and overcast skies in all cases, the lapse rate in the lower layers was largely controlled by the mechanical turbulence and was therefore quite similar in hurricane and non-hurricane cases. The variation of momentum flux from situation to situation must therefore be produced largely by the difference in vertical profiles of stress, because the surface roughness is unchanged and the kinetic energy contained in the turbulent eddies (approximately proportional to V^2 and inversely proportional to static stability) must lie within a small range.

The March cases were included so that straight flow could be compared to the highly curved flow of hurricanes under the same conditions of roughness

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[2]For convenience the term $\frac{1}{\rho}\frac{\partial \mathcal{T}}{\partial z}$ will be referred to as the "friction term" and will be written as the total vector $\mathbb{F}$ and the components as F_t and F_n and

$$|\mathbb{F}| = \sqrt{F_t^2 + F_n^2}$$

Table 1-A. — Terms of horizontal equations of motion for three hurricanes (c.g.s.units).

Radial Distance (Deg. of Lat.)	(a) $\frac{dV}{dt}$	(b) $\frac{1}{\rho}\frac{\partial p}{\partial s}$	(a+b) F_t	(c) $\frac{v^2}{R}+$	(d) fV	(e) $\frac{1}{\rho}\frac{\partial p}{\partial n}$	(c+d+e) F_n
HAZEL – 1954							
1.0	.016	-.150	-.134	.064	.125	-.600	-.411
1.5	-.021	-.136	-.157	.033	.115	-.508	-.360
2.0	-.042	-.132	-.174	.030	.108	-.446	-.308
2.5	-.052	-.130	-.182	-.002	.100	-.403	-.305
3.0	-.052	-.121	-.173	-.007	.094	-.362	-.275
3.5	-.043	-.119	-.162	.003	.087	-.326	-.236
4.0	-.024	-.115	-.139	.028	.082	-.301	-.191
CONNIE – 1955							
1.5	.065	-.590	-.525	.119	.167	-.560	-.274
2.0	.052	-.453	-.401	.140	.136	-.469	-.193
2.5	.012	-.336	-.324	.085	.103	-.384	-.196
3.0	.006	-.227	-.221	.054	.072	-.285	-.159
3.5	.011	-.156	-.145	.020	.048	-.211	-.143
4.0	.010	-.114	-.104	-.011	.028	-.169	-.152
DIANE – 1955							
1.0	-.028	-.271	-.299	.111	.096	-.556	-.349
1.5	-.011	-.271	-.282	.019	.092	-.499	-.388
2.0	-.002	-.241	-.239	.052	.084	-.418	-.282
2.5	.001	-.195	-.194	.031	.074	-.318	-.213
3.0	-.003	-.155	-.158	.025	.063	-.243	-.155
3.5	-.001	-.136	-.137	.002	.054	-.205	-.149
4.0	.007	-.115	-.108	-.010	.049	-.168	-.129

Table 1-B. - Terms of horizontal equations of motion for three non-hurricane cases (c.g.s. units).

Distance	(a) $\frac{dV}{dt}$	(b) $\frac{1}{\rho}\frac{\partial p}{\partial s}$	(a+b) F_t	(c) $\frac{V^2}{R}$	(d) rv	(e) $\frac{1}{\rho}\frac{\partial p}{\partial n}$	(c+d+e) F_n
MARCH 25, 1947							
0	-.069	-.070	-.139	.057	.120	-.149	.028
0.5	-.044	-.075	-.119	.052	.116	-.147	.021
1.0	-.013	-.107	-.120	.027	.116	-.185	-.042
1.5	-.009	-.121	-.130	.048	.097	-.186	-.041
2.0	-.023	-.120	-.143	.042	.036	-.177	-.049
2.5	-.026	-.092	-.118	.018	.074	-.142	-.050
3.0	-.009	-.105	-.114	-.004	.086	-.139	-.057
3.5	-.015	-.104	-.119	-.021	.090	-.123	-.054
4.0	-.003	-.109	-.112	-.030	.094	-.129	-.065
4.5	.008	-.057	-.049	-.008	.101	-.175	-.082
MARCH 27, 1950							
0	-.008	-.130	-.138	.006	.065	-.185	-.114
0.5	-.017	-.169	-.186	.013	.055	-.208	-.140
1.0	.011	-.130	-.119	-.003	.060	-.205	-.148
1.5	.025	-.097	-.072	-.031	.086	-.227	-.172
2.0	.028	-.079	-.051	-.019	.103	-.244	-.160
2.5	.019	-.075	-.056	±.000	.115	-.246	-.131
3.0	.034	-.096	-.062	.023	.130	-.233	-.085
3.5	-.042	-.112	-.154	.032	.121	-.210	-.057
4.0	-.072	-.137	-.209	.028	.071	-.195	-.096
MARCH 11, 1952							
0	-.004	-.087	-.091	-.007	.071	-.157	.093
0.5	.008	-.106	-.098	.005	.075	-.164	.084
1.0	.010	-.118	-.128	.001	.075	-.163	.087
1.5	-.018	-.130	-.148	-.013	.086	-.161	.088
2.0	-.036	-.133	-.171	±.000	.090	-.159	.069
2.5	-.030	-.133	-.163	-.003	.099	-.159	.063
3.0	-.033	-.157	-.190	-.014	.098	-.157	.073
3.5	-.011	-.157	-.168	-.015	.094	-.157	.078
4.0	.021	-.139	-.118	-.032	.098	-.163	.097

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and stability. Surface maps for about nine years were inspected and three cases selected where the wind speed was in the proper range and the flow was straight. In spite of this selection, it was found that curvature of the trajectories was frequently significant, so it was necessary to compute the curvature term for every case, both hurricane and non-hurricane.

In the interest of obtaining the best possible estimate of each term of the equations the following method of analysis was adopted:

1. All substantial derivatives were expanded into their partial derivative equivalents which were evaluated from the analysis, thus avoiding the assumption of steady state.

2. Cross-isobar angle of flow was measured by analyzing independently isogons of the streamlines and isogons of the geostrophic wind and subtracting those fields. This avoided the questionable assumption of circular isobars or the uncertain technique of measuring angles between two intersecting curves.

3. The cyclostrophic term, V^2/R, was evaluated by computing the <u>trajectory</u> curvature as follows:

$$\frac{1}{R} = \frac{d\alpha}{ds} = \frac{\partial\alpha}{\partial s} + \frac{\partial\alpha}{\partial t}\frac{dt}{ds} = \frac{\partial\alpha}{\partial s} + \frac{1}{V}\frac{\partial\alpha}{\partial t}$$

therefore,

$$\frac{V^2}{R} = -V^2\frac{\partial\alpha}{\partial s} - V\frac{\partial\alpha}{\partial t}$$

(where the negative sign is introduced because the meteorological convention of measuring azimuth is the negative of the mathematical one). This side-steps the difficult task of measuring the curvature of a streamline which changes from point to point and eliminates the error of assuming that stream-line and trajectory are identical.

Incorporating the above into equations (2) and (3) gives:

$$F_t = \frac{\partial V}{\partial t} + V\frac{\partial V}{\partial s} + \frac{1}{\rho}\left|\nabla p\right|\sin\beta \tag{4}$$

$$F_n = -V^2\frac{\partial\alpha}{\partial s} - V\frac{\partial\alpha}{\partial t} + fV + \frac{1}{\rho}\left|\nabla p\right|\cos\beta \tag{5}$$

where, $w\frac{\partial V}{\partial z}$ is assumed zero;

α = direction of the wind,
f = Coriolis parameter,
∇p = total horizontal gradient,
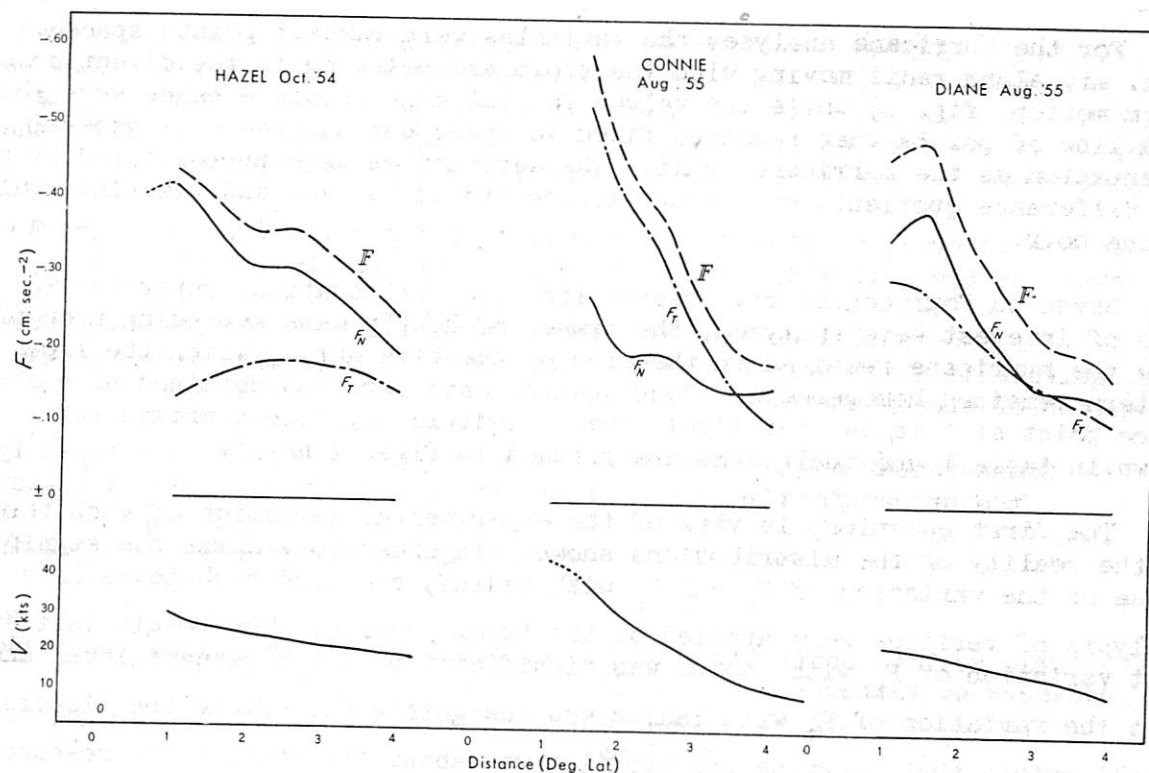β = angle between isobars and streamlines.

Figure 2. - (Upper) Profiles of total friction and its normal and tangential components for three hurricanes. (Lower) Wind speed.
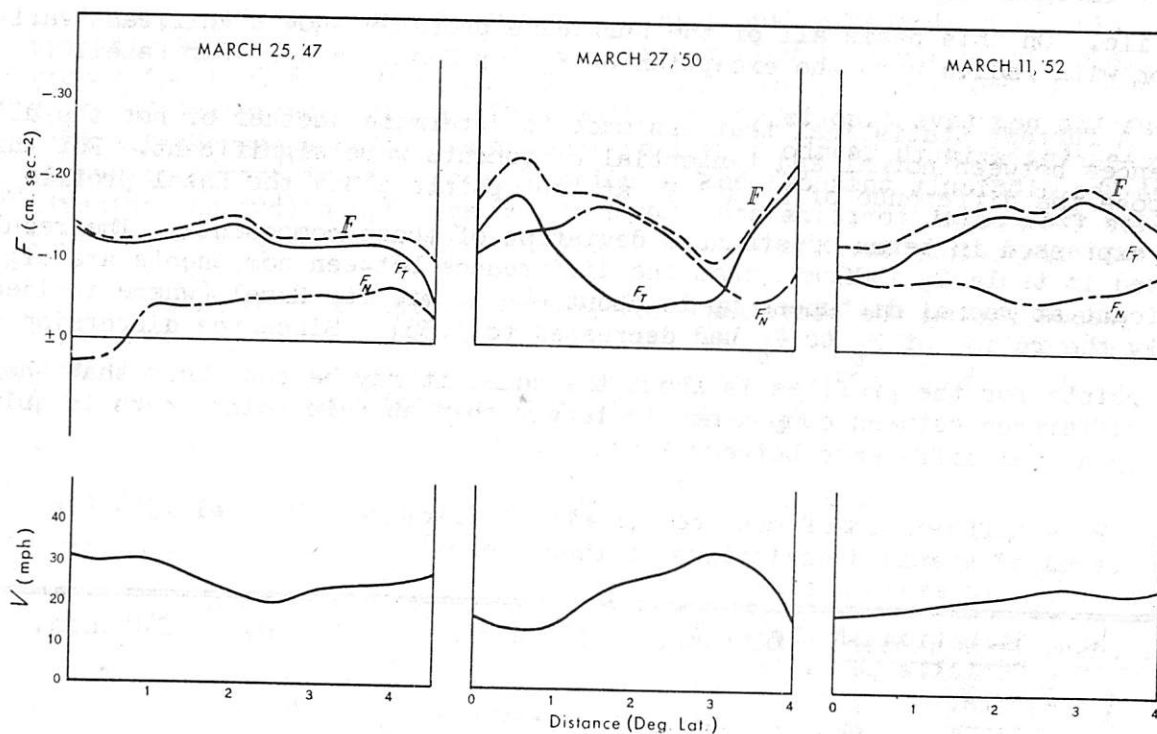

Figure 3. - (Upper) Profiles of total friction and its normal and tangential components for three non-hurricane cases. (Lower) Wind speed.

For the hurricane analyses the variables were read at points spaced 30 n. mi. along radii moving with the storm and pointing in the direction of storm motion (fig. 1) while the values for the non-hurricane cases were read at a line of points that remained fixed in space but had the same general orientation as the hurricane radii. The derivatives were approximated by finite difference quotients over space increments of 100 km. and time intervals of one hour.

Seven to fourteen hourly observations from all stations reporting in the area of interest were analyzed, the number of hourly maps depending upon the time the hurricane remained in the area or the time during which the flow pattern remained homogeneous. Consequently each term was obtained at each space point at 7 to 14 individual times. Individual, time-averaged terms are shown in table 1 and their sums are graphed in figs. 2 and 3.

The first question, in view of the experimental variation of each term, is the reality of the distributions shown. In order to examine the significance of the variation of F_t and F_n with radius, standard techniques of analysis of variance were applied to the Hazel profile. The result indicated that variation of F_n with radius was significant on the 5 percent level and that the variation of F_t with radius was insignificant. Since the dispersion of all points that comprise the profiles was about the same, it is reasonable to regard change of friction with radius as significant where it has a variation of the order of the F_n-profile for Hazel and to regard it as statistically insignificant where it has a variation with radius as small as the F_t-profile. On this basis all of the hurricane profiles show significant variation with radius with the exception of F_n for Connie and F_t for Hazel.

A further statistical test was made to determine whether or not the differences between normal and tangential components were significant. For this purpose the difference of F_t and F_n at four points along the Hazel profile was expressed in terms of standard deviation of those components. The results, listed in table 2, indicate that the differences between components are significant at radial distances up to about 240 n. mi. for Hazel (where incidentally the ratio of F_t to F_n had decreased to 1.35). Since the dispersion of all points for the profiles is about the same, it may be concluded that where the difference between components is larger than at this point there is quite likely a real difference between F_t and F_n.

Table 2. - Differences of mean components of friction for Hazel 1954 (in terms of standard deviations of those means).

r (dis. from center)	60 n.mi.	120 n.mi.	180 n.mi.	240 n.mi.
$\dfrac{F_t - F_n}{\sigma_{\bar{x}}}$	6.0	4.0	3.4	2.2

One feature of the analysis that varies among the individual cases is the different motion of the coordinate systems. Errors that might result were minimized by avoiding the steady state assumption, but it is of interest to know whether or not the radial profiles of friction changed in a systematic way as the coordinate systems were carried northward. For this reason radial profiles of friction were computed for ten individual hours for Diane from the time the center crossed the coast until it had moved about 100 n. mi. For the first eight of these ten hours the significant features of the profile remained unchanged; during the last two hours the profile flattened somewhat. Five in-

Figure 4. - Total friction as a function of speed squared.

dividual hourly profiles were also computed for March 1950 (a fixed coordinate system) and again the space profiles remained nearly constant in time, therefore the moving coordinate system apparently did not distort the comparisons discussed in the following sections.
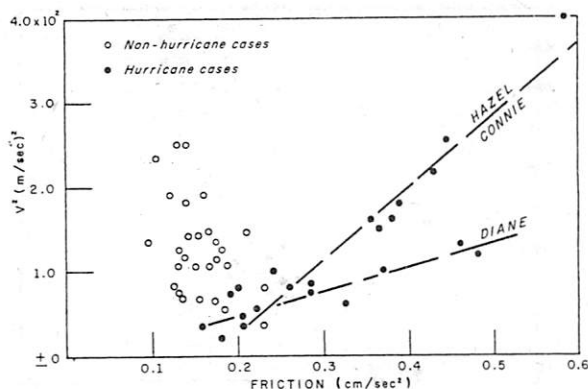
Friction as a function of speed

In a large variety of investigations where the details of the boundary layer are important, magnitude of the frictional effect is critical. As a specific example, the angle of cross-isobar flow in a steady-state system is proportional to the friction, for the downstream pressure gradient must be just sufficient to overcome deceleration by friction. Thus the divergence (and all the sequence of events that follow) produced by differential cross-isobar transport is in turn a function of the frictional effect. While the circulations studied here are not in steady state, time changes are small so that the same general effect obtains.

To estimate the frictional terms of equations (2) and (3) where no direct data exists, meteorologists have applied to equation (1) the assumption that the vertical shear vector is oriented in the direction of, and is proportional to the wind itself; that is,

$$\frac{\partial V}{\partial z} = k'V$$

where k' is the proportionality factor.

For a small height range where density, drag coefficient, and wind direction are constant with height, the frictional terms which appear in component form as the last terms of (2) and (3) can be written:

$$|F| = \frac{1}{\rho}\frac{\partial}{\partial z}\left(\rho\, c_d^2\, v^2\right) = c_d^2\, V\frac{\partial V}{\partial z} \quad ;$$
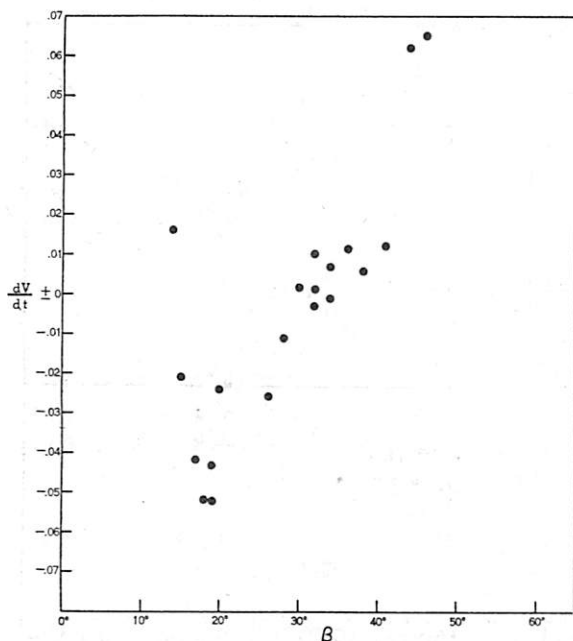
and, introducing the assumption mentioned above,

Figure 5. - Acceleration (cm.sec.$^{-2}$) as function of cross-isobar flow in degrees.

$$|F| = k V^2 \qquad (6)$$

where $k = k'c_d^2$, a constant.

The results of this analysis can be examined to determine just what proportionality constant k applies to equation (6). Figure 4 is a plot of $|F|$ versus V^2. It is apparent that while these quantities in each hurricane are roughly proportional, the "constant k" is different for each storm. No such relation is valid for the non-hurricane cases, however.

The point here appears to be, not what proportionality constant to use, but rather that in general, no such relationship applies. This is really not surprising when one considers the basis for equation (6). The importance of the normal component of friction (fig.2) suggests that the shear vector must be oriented different from the anemometer wind direction, contrary to the assumption involved in equation (6).

On the other hand, it is reasonable to expect a large component of the wind throughout the lowest few hundred meters to be directed along the s-axis (direction of anemometer level wind), therefore if the shear of stress is in fact proportional to the shear of wind, the tangential component of friction should be roughly proportional to the square of the surface wind. A plot of F_t versus V^2 however, failed to reveal a unique proportionality constant and for Hazel the points fell along a curve with both positive and negative slope.

In light of the foregoing, it is necessary to conclude that frictional drag on the boundary layer over surfaces of the same roughness is not a unique function of wind speed.

Friction in relation to cross-isobar flow

Despite the conclusion just stated, some relation between surface wind and vertical shear of stress is revealed for the hurricane cases, albeit a relation that differs from case to case. Consequently one is prompted to examine the other variables of this analysis for a clue as to why this should be so.

Since the pressure gradient terms are the dominant factors in determining the magnitudes of the respective components of friction (see table 1) the angle of cross-isobar flow is critical because it reflects what part of the total pressure gradient is directed along, and normal to, the fluid motion.
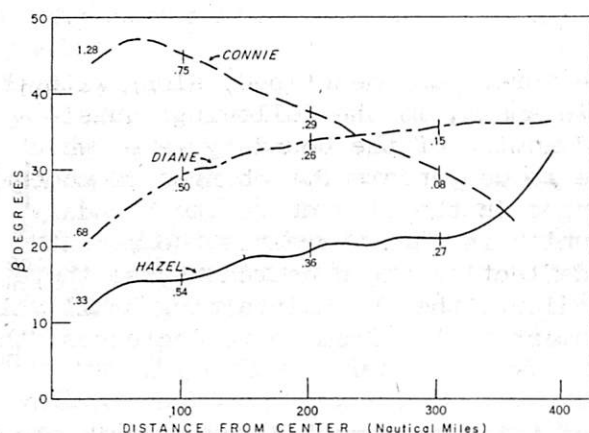
Figure 6. - Cross-isobar angle (degrees) for three hurricanes with pressure gradient per unit mass shown in units of cm. sec.$^{-2}$.
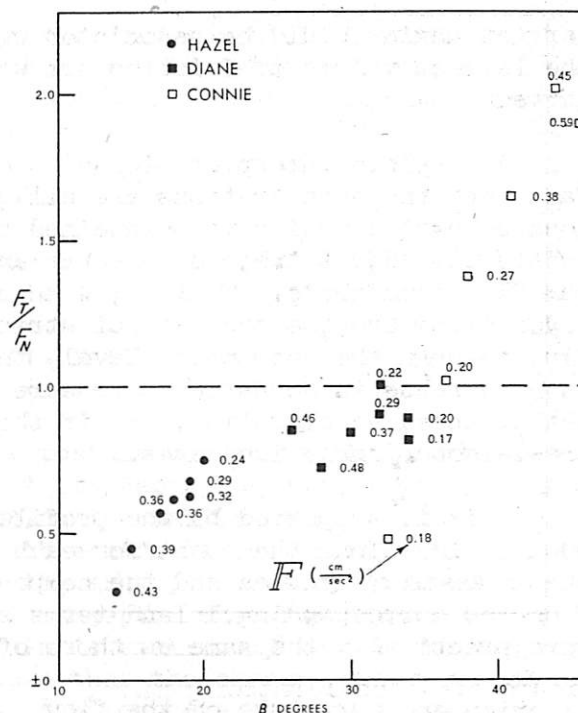

Figure 7. - Ratio of frictional components as function of cross-isobar flow, with magnitude of total friction shown at each point.

Physical reasoning suggests that the larger deflection angles might be associated with larger downstream pressure force and thereby with accelerated motion, which in turn might have a vertical profile of speed quite different from the nonaccelerated cases. Examination of the data revealed almost a complete lack of correlation between acceleration and pressure force; only two points for one storm had large accelerations associated with large downstream pressure. On the other hand a definite relation between acceleration and deflection angle does appear and is illustrated in figure 5, suggesting that the increment of momentum injected into the anemometer level flow to produce acceleration has a large cross-wind component. If real, this is an interesting feature for it implies that events in the upper layers control the acceleration and convergence of the surface layer, rather than the cross-isobar flow representing a simple reaction between surface roughness and pressure gradient. Further support to this is offered by figure 6 that shows how the cross-isobar flow is different from storm to storm, as well as the pressure gradient which shows no systematic relation to the angle.

Figure 7, which is shown to illustrate the relation between cross-isobar flow and the partition of total friction into its components, includes the values of total friction at each point. Again, if the deflection angle were simply due to the fact that increased surface drag decelerated the air thus enabling the pressure gradient to deflect the flow across the isobars, the

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largest angles would be associated with the largest values of friction, but the largest values of friction are well distributed from the smallest to the largest angles.

A possible interpretation of the features just described, along with the fact that the accelerations are all quite small, is the following; quasi-nonaccelerated motion is maintained by dynamics of the boundary layer which constantly adjust the cross-isobar angle to compensate for changes of momentum flux from above. That is, when changes in the flow above the boundary layer bring about a decrease of stress which in turn increases the momentum flux through the anemometer level, the deflection angle decreases, so there is a decrease in downstream pressure gradient, thereby maintaining small values of speed acceleration, and if the momentum flux from above decreases, the cross-isobar angle increases.

This is supported by the profiles of friction computed for a strip about 50-km. wide along the coast for each of the hurricanes (not shown). Although the cross-isobar angles and the components of friction were quite different from the corresponding inland terms shown here, the total friction profiles were practically the same as those of figure 2.

Friction and curvature of the flow

Magnitude of total friction in hurricanes was about double that of the non-hurricane cases. The former showed total friction up to 0.4 cm. sec.$^{-2}$ while the non-hurricane cases gave something less than half - up to 0.17 cm. sec.$^{-2}$. The relative importance of the normal component of friction, especially in hurricanes, raises the question as to what feature of hurricane flow is so different. An obvious difference is curvature of flow, and if this is somehow connected with larger values of friction one would expect it to be manifest through the curvature term (V^2/R) in equation (3); the three spring cases provided a means of comparing flow of different curvature. The curvature terms for hurricanes had a large range which both exceeded and overlapped the range of values for non-hurricane flow.

Since the cyclostrophic term (V^2/R) appears in the equation for the normal component, but not for the tangential component of friction, it would seem reasonable that it might exercise an important control on the magnitude of F_n. A plot of F_n versus V^2/R revealed no such relation however. A slight relation between total friction F and V^2/R did appear but the correlation was insignificantly low. Due to the fact that the acceleration terms are interrelated there is the possibility that linear correlations between pairs are obscurred by variations in a third term. In order to examine that possibility, values of cross-isobar flow, radius of trajectory curvature, and speed were successively plotted in the graphs just discussed. If multiple correlation exists among three of these variables it would have been shown by the pattern of isolines drawn for various parameters, but none could be discerned. Therefore insofar as these data are concerned, no correlations could be detected.

The implication of this examination is that the centrifugal term is of minor importance in the range of values studied here. It should be noticed

however that maximum winds were only 20 m.p.s. and the minimum radius of curvature 200 km., even at distances 100 km. from the storm center, so the highly curved flow near the maximum wind zone of hurricanes has not been studied.

3. DEPTH OF THE INFLOW LAYER

Some clue concerning the depth of the inflow layer and its variation with distance from the storm center might be obtained from surface data alone if the following simplified model resembles nature, and this in turn is of interest because of its connection with friction and cross-isobar flow in the boundary layer.

The simplified model is based on the postulate that the inflow layer is characterized by divergent flow throughout its depth but at the top of the layer, where the influence of surface stress is small, the flow is non-divergent. The total precipitation in such a circulation would be directly related to the net water vapor flux into the area produced by the convergence in the boundary layer, for it is generally accepted that the flux of water vapor away from a hurricane at upper levels is small compared to the flux inward at low levels. It has also been shown that the change over a period of an hour in the water storage in a column of atmosphere that is largely cloud-filled, is small compared to the total precipitation that occurs during that hour with moderate rainfall.

Under these conditions:

$$I = \frac{1}{A} \int_A \int_0^h \nabla \cdot (\rho q \, \mathbf{V}) \, dz \, dA \tag{7}$$

where: I = mass of precipitation per unit time per unit area,
   ρ = density of air,
   q = specific humidity,
   $\mathbf{V}$ = horizontal wind,
   A = area for which divergence is measured,
   h = height of lowest level of non-divergence.

Integration of equation (7) requires a knowledge of the divergence of $(\rho q \, \mathbf{V})$ throughout the layer. Examination of soundings indicates that for our purposes, ρq can be represented by a constant for the lowest 1.5 km. and by a different constant if approximately 3.0 to 3.5 km. is involved. (This simplification is not valid for greater heights). As a first approximation then, the integration may be performed if the profile of velocity divergence can be estimated.

It is quite certain that the convergence in the boundary layer decreases with height in a complicated fashion, depending upon mechanical turbulence, the degree of penetrative convection, and how the wind speed changes with height. Suppose the actual divergence profile were the one illustrated schematically in figure 8; the flux of water vapor into the area would be proportional to the area under the curve, up to the first level of non-divergence. Now the same flux of water vapor would be obtained if the profile were linear, with the same value as the actual profile at anemometer
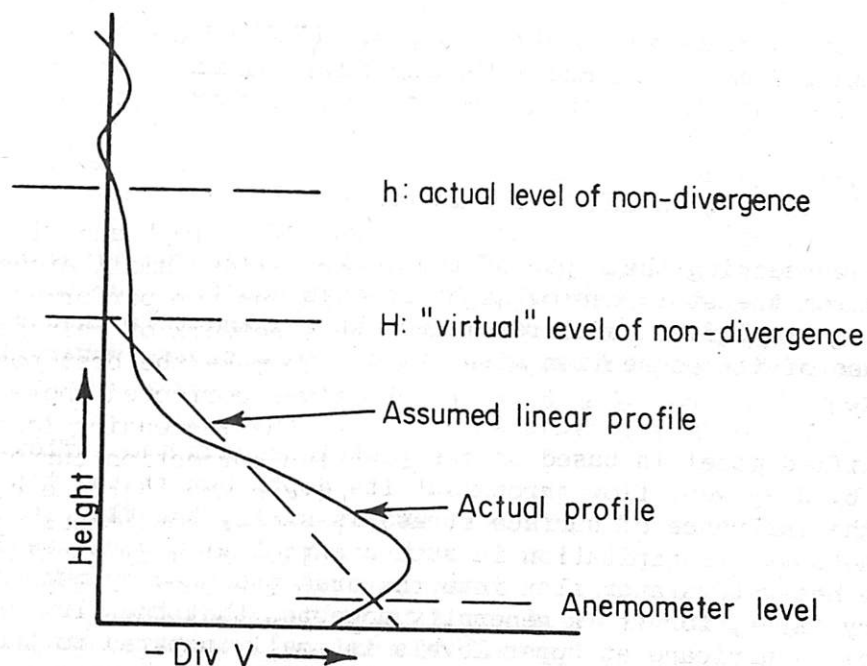
Figure 8. - Hypothetical profile of divergence and levels of nondivergence.

level and a level of non-divergence at some virtual height, H. It is clear that the virtual height H would be quite different from the actual height h, but it also appears likely that any important variation of h with distance from the storm center would be reflected in changes of the virtual height as well.

Using this simplification, equation (7) may be integrated and written in terms of averages for the column under consideration:

$$I = \overline{\nabla \cdot V} \; \overline{(\rho q)} \; H$$

This equation will yield a slight overestimate of the rainfall intensity because the flux away from the area A at upper levels is ignored. Now let D designate the mean divergence in the inflow layer and D_o its value at anemometer level, and let $Q = \frac{\partial}{\partial z} (\rho q)$, which is a constant once H is approximated, then,

$$D = \frac{D_o}{2} \quad \text{and} \quad \overline{(\rho q)} = \rho_o q_o + \frac{H}{2} \frac{\partial}{\partial z} (\rho q)$$

$$\therefore I = \frac{D_o}{2} \left[(\rho_o q_o) + \frac{HQ}{2} \right] H$$

Solving for H,

$$H = -\frac{\rho_o \, q_o}{Q} \pm \sqrt{\left(\frac{\rho_o \, q_o}{Q}\right)^2 + \frac{4I}{QD_o}} \qquad (8)$$

Method of analysis

Isohyetal maps for each hour were drawn for Hazel 1954 and Diane 1955 and the total rainfall for each of the hours for various small areas was computed by graphical integration. Similar analysis was not performed for Connie 1955 because the storm path was not through a network of rain gages adequate to obtain hourly precipitation analysis. To make the best possible estimate of rainfall rates, each station report was corrected where necessary. The corrections, an attempt to take into account the increasing loss of catch in high winds, were made by applying either of two correction curves constructed from what information is available in the literature [4]; one curve for heavy rain (which presumably occurred under convective conditions with large drops) and another for moderate and light rainfall. At many stations the corrections amounted to less than .01 inch during the hour, and no correction was applied, but at a few they amounted to adjustments as great as 0.10 inch. Some of the stations have storage gages but by using the technique of cumulative mass rainfall curves at stations where hourly amounts were reported, the cumulative 6-hourly reports could be divided quite satisfactorily into hourly amounts.

Divergence was computed by reading velocities on a grid with 50-km. spacing, from maps which had been analyzed for the middle of each rainfall accumulation period. Subsequently, overlapping computations of divergence were space-averaged for the same area for which rainfall had been integrated, and other appropriate values were read from surface analysis for use in equation (8).

In general the computations involved the forward and left quadrants of the storms and analysis was restricted to areas well removed from the foothills and mountains of the central Carolinas and Virginia. Near the hurricane center the gradient of divergence was so large compared to the grid spacing that no dependable computations could be made.

Results of precipitation analysis

Figure 9 shows the virtual heights H versus distance from the center ahead of the storms. Each of the values represents time averages and the arrows indicate the magnitude of plus and minus one standard error of the computed means (i.e., $S \, E_H = \frac{\sigma_H}{\sqrt{N}}$).

In view of the simplifying assumptions and the experimental error involved, the results are of course rather crude, but the inflow layer does appear to have a consistent change of depth with radius; the inflow layer (ahead of the storm) decreases toward the hurricane center. Since this analysis does not consider the zone of maximum wind and maximum convective activity, this apparent decrease is not impossible and may well reflect a real characteristic
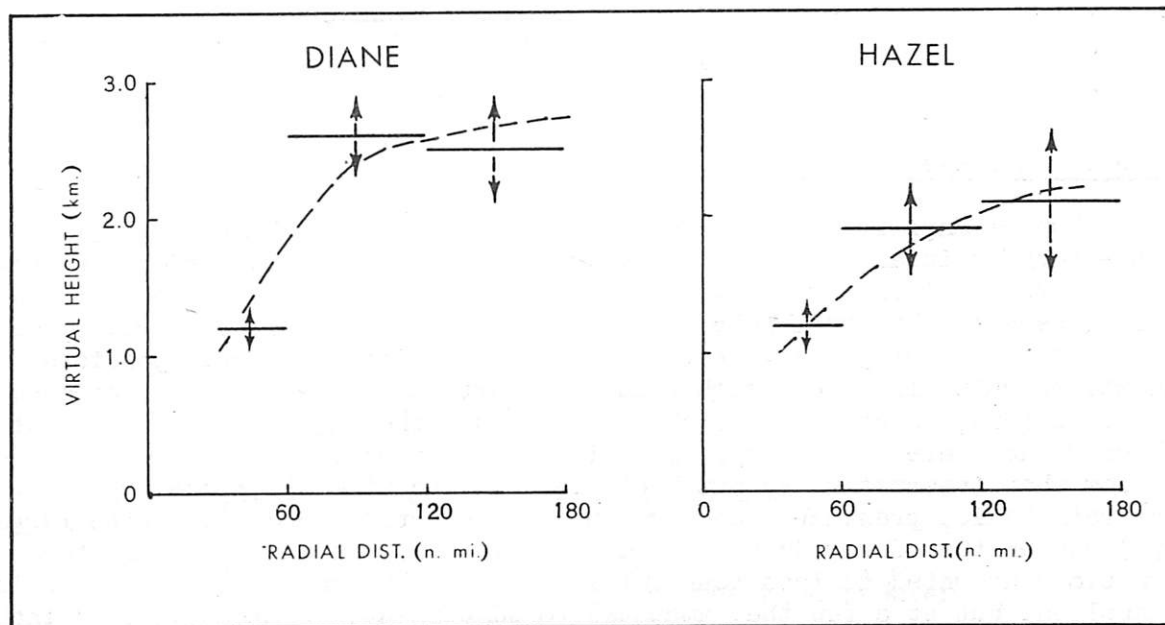
Figure 9. - Virtual height of inflow layer as function of radial distance from center. (Arrows indicate ⁺⁄₋ one standard error of mean.)

(e.g., see [2]). The important result of this moisture flux analysis from the standpoint of friction, however, is due to the places where the simplified model **fails** rather than where it "succeeds."

In hurricanes where the convective activity is large, the so-called friction layer must extend to very great elevations and the inflow must also occur to some degree at great heights. Now it must be borne in mind that the "virtual height" computation depends upon the assumption of a linear profile of divergence and it is possible that the "decreasing height" could be brought about by a change in the shape of the profile even though the real height of the nondivergent level was unchanged, or even increased - it is only necessary that the area under the curve decrease with the other parameters remaining constant. Indeed there is evidence that such a change does occur when the nature of the underlying surface is changed, and it is this result that provides additional insight into the boundary layer characteristics.

Figure 10 shows the virtual height and anemometer level convergence as a function of time in a sector of Diane 30 to 60 n. mi. from the center in the forward quadrant during the period in which this sector crossed the coast and moved inland. The virtual height decreased by a factor of two from 0530 to 1230 EST while the convergence showed a variation of only 10 percent. Since the specific humidity of the air involved was essentially constant
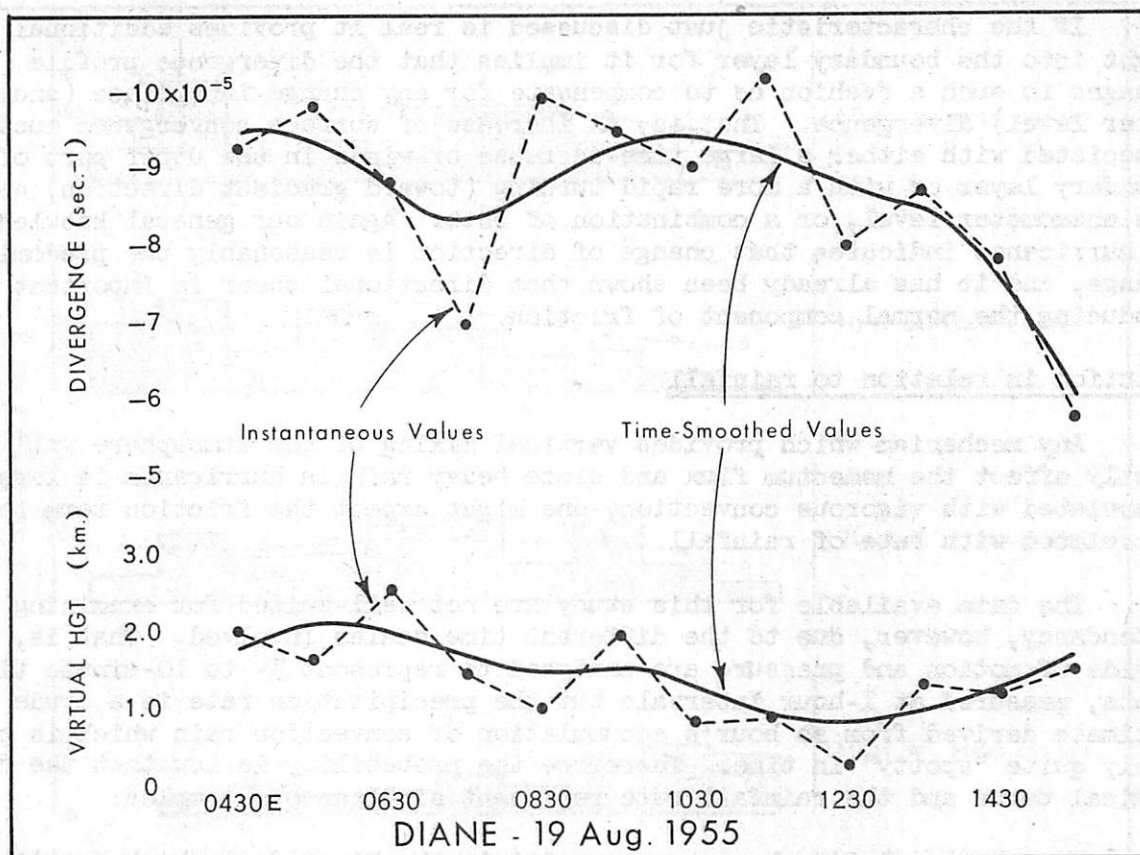
Figure 10. - Virtual height of inflow layer in Diane 1955 and surface divergence as function of time.

we must either accept as a fact that the inflow layer decreased by this large amount or conclude that the profile of convergence changed shape. The most probable explanation is the latter because on the basis of our general knowledge of hurricane structure it is quite unreasonable to believe that the inflow occured in a layer less than 1 km. thick this far outside the zone of maximum vertical motion.

A second piece of evidence indicates a variable relation between virtual height and profile of frictional inflow. It would seem that equations (2) and (3) might be integrated vertically from the surface to the top of the frictional inflow layer to provide values of surface stress by the method used by Palmén and Riehl [5]. Their technique requires that vertical averages of the various terms be used and, for lack of any indication to the contrary, it would be necessary here to assume for the various terms the same relation between anemometer level and average value at all radial distances from the storm center. The fact that this yielded completely unreasonable results indicates that quite likely stress does not vanish at the virtual height of the inflow layer and therefore the "virtual top" of the inflow layer is not the top of the friction layer.

If the characteristic just discussed is real it provides additional insight into the boundary layer for it implies that the divergence profile changes in such a fashion as to compensate for any change in surface (anemometer level) divergence. That is, an increase of surface convergence must be associated with either a large time-decrease of winds in the upper part of the boundary layer or with a more rapid turning (toward gradient direction) above the anemometer level, or a combination of both. Again our general knowledge of hurricanes indicates that change of direction is reasonably the predominant change, and it has already been shown that directional shear is important in producing the normal component of friction.

Friction in relation to rainfall

Any mechanism which provides vertical mixing of the atmosphere will directly affect the momentum flux and since heavy rain in hurricanes is largely associated with vigorous convection, one might expect the friction term to be correlated with rate of rainfall.

The data available for this study are not well-suited for examining such a tendency, however, due to the different time scales involved. That is, the fields of motion and pressure are analyzed to represent 5- to 10-minute time-means, measured at 1-hour intervals but the precipitation rate is a crude estimate derived from an hour's accumulation of convection rain which is probably quite "spotty" in time. Therefore the probability is low that the dynamical terms and the rainfall rate represent simultaneous samples.

Some such relation however may exist in the Diane data which are illustrated in figure 11. This figure shows the total friction term at various radial distances from the center at 30 minutes past each of ten hours, along with the precipitation accumulated during the hour. Notice that at distances of 1.0, 1.5, and 2.0 degrees of latitude from the center, where the rainfall amounts are largest, there is a slight tendency for the friction and precipitation to change in the same sense, but due to the reasons mentioned above, this line of inquiry was not pursued.

4. SUMMARY

The work reported here should be viewed as an experiment in the application of surface analysis and case-study technique as well as a study of friction in hurricanes. Certain results concerning the former are definitive but conclusions relating to friction are largely inferential and therefore leave something to be desired. Furthermore, the portion of the hurricane flow investigated was completely over land, the pressure profiles had already been modified, and the wind speeds were less than hurricane force. Therefore the results cannot be taken as general hurricane characteristics, rather they should be interpreted as applying to this particular phase of the hurricane life cycle. The results are important in that they indicate the nature of the problem and point up differences that must exist between hurricane and non-hurricane flow, although an important question left open is why the non-hurricane flow exhibits quite different characteristics. While it is true that the hurricane flow is in general more curved than that of the non-hurricane cases, it has been shown that the difference in friction is only in small part associated with curvature.
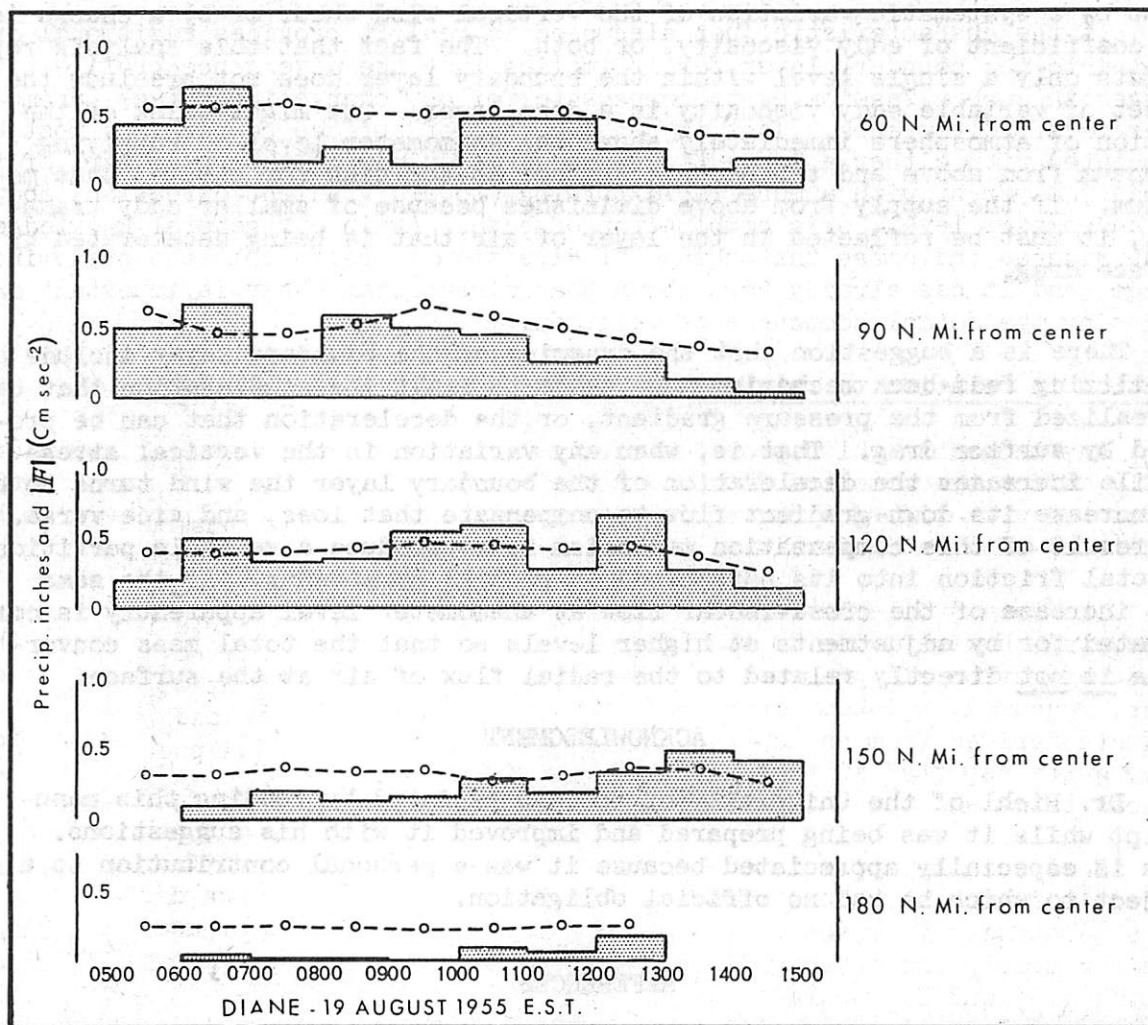
Figure 11. - Hourly precipitation and total friction as function of time for Diane 1955.

One result is a warning to hurricane workers not to make a wide and general application of boundary layer characteristics deduced from non-hurricane experiments.

A primary and firm result is the demonstration that friction to which the flow is subject increases toward the storm center, but that it is not a unique function of wind speed or of surface roughness. Secondly, the component of friction normal to the wind direction is significantly large and may exceed the tangential component at anemometer level over land. This however, cannot be generalized to over-water flow of hurricanes.

Equally certain is the fact that the vertical profile of stress varies with distance from the center, but it is not known whether this is brought

about by a systematic variation of the vertical wind shear or by a change in the coefficient of eddy viscosity, or both. The fact that this analysis represents only a single level within the boundary layer does not preclude the effect of variable eddy viscosity in a deep layer. One might think of the portion of atmosphere immediately above the anemometer level as supplying momentum from above and the earth's surface as the sink for part of that momentum. If the supply from above diminishes because of smaller eddy transport, it must be reflected in the layer of air that is being decelerated by surface drag.

There is a suggestion that the dynamics of the boundary layer include a stabilizing feed-back mechanism that tends to limit the acceleration that can be realized from the pressure gradient, or the deceleration that can be produced by surface drag. That is, when any variation in the vertical stress profile increases the deceleration of the boundary layer the wind turns so as to increase its down-gradient flow to compensate that loss, and vice versa. The result of this compensation mechanism is to produce a variable partition of total friction into its normal and tangential components. At the same time increase of the cross-isobar flow at anemometer level apparently is compensated for by adjustments at higher levels so that the total mass convergence is not directly related to the radial flux of air at the surface.

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